

CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2013
Individual Medley, Junior - Senior Level

For each of the following questions, blacken the appropriate circle on the answer sheet. Each correct answer is worth four points. **One point is deducted for each incorrect answer.** An unanswered question is given zero points. Note that random guessing may adversely affect your score.

You have 50 minutes to complete the examination. If you finish early, review your answers. When the exam is over, give your answer sheet to the proctor.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. Let $f(x) = x/3$ if x is a multiple of 3 and $f(x) = 2x + 1$ otherwise. Determine $f(f(f(100)))$.

- A. 210 B. 135 C. 99
D. 32 E. 96

2. Points A, B, C, D, E, F are such that A, B, C are collinear. Likewise, D, E, F are also collinear (on a different line). How many distinct lines are determined by these six points?

- A. 11 B. 9 C. 15
D. 12 E. 6

3. A square is inscribed in a unit square. One corner of the inner square is $1/4$ away from the corner of the outer square. The length of the diagonal of the inner square is $\sqrt{b/4}$. Solve for b . [this question edited after Math Field Day]

- A. 2 B. 3 C. 4
D. 5 E. 6

4. A chord of a circle has length 6 and is 1 unit away from and parallel to a diameter of the circle. What is the length of the diameter?

- A. 7 B. $\sqrt{20}$ C. 8
D. $\sqrt{10}$ E. $2\sqrt{10}$

5. What is the value of the 4th degree coefficient of $(x+1)(2x+1)(3x+1)(4x+1)(5x+1)$

- A. 274 B. 304 C. 154
D. 194 E. 234

6. Let $f(x) = \frac{3x+2}{4x-3}$. Find $f^{-1}(x)$.

A. $\frac{3x+2}{4x+3}$

B. $\frac{3x-2}{4x-3}$

C. $\frac{3x-2}{4x+3}$

D. $\frac{3x+2}{4x-3}$

E. $\frac{4x-3}{3x+2}$

7. Given that x, y, z are distinct positive integers, what is the smallest possible value for

$$\frac{x}{x+1} + \frac{y}{y+1} + \frac{z}{z+1} ?$$

A. 5/6

B. 3/2

C. 13/12

D. 23/12

E. 49/24

8. Given that the points (0,4) and (4,1) are on the graph of $f(x) = Ce^{kx}$, find $f(6)$.

A. 2

B. 1

C. 1/2

D. 1/4

E. None of these

9. A $3 \times 3 \times 3$ cube is painted on 3 mutually adjacent faces. The cube is then cut into 27 equal $1 \times 1 \times 1$ cubes. One of these smaller cubes is chosen randomly. It is rolled, as a die. What is the probability that the top of the rolled cube is painted?

A. 1/3

B. 1/6

C. 2/9

D. 3/11

E. 7/18

10. Seven vampires and eight werewolves met in a forest. During the meeting, a grizzly bear jumped out and randomly mauled three of them. What is the probability that all three mauling victims were vampires?

- A. $1/13$ B. $7/15$ C. $7/150$
D. $1/15$ E. $3/26$

11. Ten table tennis players play a tournament. No match ends in a tie. A player is eliminated from the tournament after his or her second loss. The tournament is over when only one player remains. Let x and y be the maximum and minimum possible number of matches before a winner is determined. Then $x+y = ?$

- A. 35 B. 36 C. 37
D. 38 E. 40

12. In a given triangle, a line through the centroid and parallel to one of the sides determines a smaller triangle and a trapezoid. What is the ratio of the area of the small triangle to the area of the trapezoid?

- A. 1 B. $2/3$ C. $4/9$
D. $5/9$ E. $4/5$

13. All the solutions to $\frac{x}{y} + \frac{y}{x} = 3$ lie on a

- A. circle B. hyperbola C. ellipse
D. line E. pair of lines

14. A bowl has 3 red, 4 green, 5 blue, and 6 yellow marbles. If three marbles are picked randomly, what is the probability that the three marbles will be different colors?

- A. $1/2$ B. $1/3$ C. $57/136$
D. $48/221$ E. $3/4$

15. For a given isosceles right triangle, let R be the radius of the circumscribed circle and r be the radius of the inscribed circle. Then $R/r = f(\pi/8)$ if f is which function?

- A. \sin B. \cos C. \tan
D. \cot E. \sec

16. What is the area of the triangle with vertices $(0,0,1)$, $(0,1,0)$, $(1,0,0)$?

- A. $\sqrt{3}/2$ B. $1/2$ C. $1/3$
D. $\sqrt{3}/4$ E. $1/6$

17. Five cards are dealt from a standard deck that also has two jokers. How many ways are there to make a straight, using exactly one joker?

- A. 64000 B. 12000 C. 12400
D. 25600 E. 5120

18. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, \dots, 30\}$ and A_i be the multiples of i that are in U . How many elements are in the complement of $\bigcup_{i=5}^9 A_i$?

- A. 12 B. 15 C. 18
D. 20 E. 10

19. Two brothers inherit a flock of sheep, which they decide to sell. The price per sheep happens to equal the size of the flock. The money comes in a stack of \$10 bills and some change. They split the money as follows. Joey first takes a \$10 bill, then Josh takes one, and they continue alternating. When the bills are gone, Joey says "you may keep all the change." But Josh says "Unfair! You got more bills than I did." Joey says "OK, I will write you a check that will make it fair." What is the amount, in dollars, that Joey should make out the check for?

- A. 1 B. 2 C. 3
D. 4 E. 5

20. One end of a rectangular swimming pool is 10 yards wide and along this end the water is 2 feet deep. At the other end, 20 yards away, the water is 10 feet deep. The depth changes steadily. The walls are vertical. At 1 cubic foot per second, which of the following is closest to the number of hours it will take to fill the pool?
- A. 1 B. 2 C. 3
D. 4 E. 5
21. The sum of thirteen consecutive odd numbers is 299. What is the largest of these numbers?
- A. 5 B. 18 C. 23
D. 35 E. 47
22. Which of these numbers can not be written as the difference of two squares?
- A. 2001 B. 2013 C. 2014
D. 2015 E. 2016
23. Albert, Betty, Carol, and Darshan drove across the country in seven days. On days when Albert sat in back, Betty sat in front. On days when Betty sat in back, Darshan also got in back, but Carol did not. On days when Darshan sat in back, Albert or Carol sat in back too. No two days had the same set of people in the back. How many days was Carol in the back?
- A. 1 B. 2 C. 3
D. 4 E. 5
24. How many zeros are at the end of $500!$? [this question edited after Math Field Day]
- A. 100 B. 124 C. 150
D. 175 E. 200
25. A regular icosahedron is a solid with twenty congruent triangular faces. How many vertices does it have?
- A. 12 B. 15 C. 18
D. 20 E. 60