CALIFORNIA STATE UNIVERSITY, BAKERSFIELD Lee Webb Math Field Day 2013 Team Medley, Freshman-Sophomore Level

Each correct answer is worth ten points. Answers require justification. Partial credit may be given. Unanswered questions are given zero points.

You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the proctor. Multiple solutions to the same problem will invalidate each other.

Elegance of solutions may affect score and may be used to break ties.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification. 1. Fill in the following array, so that each row and column form an arithmetic sequence (e.g. a sequence in which successive terms all have the same difference):

	74		
			186
		103	
0			

- 2. Find the last three digits of the product of the first 20 prime numbers.
- 3. Two parallel lines are 10 units apart. Point A is between the lines and 3 units from the nearer one. Find the area of the largest square that has point A as one vertex and does not contain any points outside the two lines.
- 4. Ella and Daniel are preparing to play a game called "Twenty-One Stones". The 21 stones will be placed in 2 piles. Daniel and Ella will alternate turns. On his or her turn, each player is to remove as many stones as he or she wants from either pile or remove the same number from both piles. The player to take the last stone wins. Ella says since she is younger, she gets to go first. Daniel says "fine but I get to divide the stones into piles." How many stones should Daniel put into each pile so that he can guarantee that he wins the game?
- 5. A sequence of digits begins with 2,0,1,3 and continues with each subsequent digit being the last digit of the sum of the previous 4 digits. Thus the next few digits in the sequence are 6,0,0,9,5,4,8.... Will the string of digits 9988 appear in this sequence?
- 6. Prove that a convex quadrilateral can be inscribed in a circle if and only if it has opposite angles are that are supplementary.