CALIFORNIA STATE UNIVERSITY, BAKERSFIELD

Lee Webb Math Field Day 2013

Team Medley, Junior-Senior Level

Each correct answer is worth ten points. Answers require justification. Partial credit may be given. Unanswered questions are given zero points.

You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the proctor. Multiple solutions to the same problem will invalidate each other.

Elegance of solutions may affect score and may be used to break ties.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. A coin is loaded so that it flips Heads 45% of the time. However, this is the only coin that anybody can find to determine which team kicks off in the Slide Rule Bowl. Nonetheless, the referee explains a procedure for using the coin that easily satisfies both team captains that they each have a 50% chance of winning the toss. Describe such a procedure.

2. Alicia, Brian, Charlie, Darlene, Erica, Frank, and Gabrielle are either engineers or sales representatives. It is well-known that the engineers always lie and the sales representative always tell the truth. Brian and Erica are sales representatives. Charlie says that Darlene is an engineer. Alicia says that Brian says that Charlie says that Darlene says that Erica says that Frank says that Gabrielle is not a sales representative. If Alicia is an engineer, how many sales representatives are there?

3. Suppose $0 < x_1 \le x_2 \le x_3 \le \ldots \le x_n$ and $0 < y_1 \le y_2 \le y_3 \le \ldots \le y_n$. Show that $\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right) \le n \sum_{i=1}^{n} x_i y_i$.

4. Find the volume of a tetrahedron that has vertices at (1,0,0), (0,1,0), (0,0,1), (1,1,1).

5. Ella and Daniel are preparing to play a game called “Twenty-One Stones”. The 21 stones will be placed in 2 piles. Daniel and Ella will alternate turns. On his or her turn, each player is to remove as many stones as he or she wants from either pile or remove the same number from both piles. The player to take the last stone wins. Ella says since she is younger, she gets to go first. Daniel says “fine – but I get to divide the stones into piles.” How many stones should Daniel put into each pile so that he can guarantee that he wins the game?

6. If a, b,c are the side lengths of a triangle, prove that

$$abc \ge (a+b-c)(b+c-a)(c+a-b)$$