CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2014
Individual Medley, Junior-Senior Level

For each of the following questions, blacken the appropriate circle on the answer sheet. Each correct answer is worth four points. **One point is deducted for each incorrect answer.** An unanswered question is given zero points. Note that random guessing may adversely affect your score.

You have 50 minutes to complete the examination. If you finish early, review your answers. When the exam is over, give your answer sheet to the proctor.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. Mr. Gresk averaged his algebra students' tests. The twenty papers had an average score of 83. Then he found two more papers that had slipped behind his desk. The average with these new papers was 77. What is the average score of the two papers that were behind the desk?

A. 6 B. 7 C. 17

D. 23 E. 71

2. What is the value of x in the following equation: $\frac{5x+6}{7x+8}=\frac{9}{10}$

A. 1 B. 3/4 C. -7/8

D. -12/13 E. 24/25

3. Put in lowest terms: $\frac{6!}{6^6}$

A. $\frac{5}{324}$ B. $\frac{5}{108}$ C. $\frac{10}{243}$

D. $\frac{10}{81}$ E. $\frac{60}{343}$

4. What is the area of a right triangle that has hypotenuse of length 10, and one angle measuring 20^o

A. $50\sin^2 20^o$ B. $50\sin 20^o \cos 20^o$ C. $\frac{100\sin 20^o \cos 20^o}{3}$

D. $25\sin 40^o$ E. $50\sin^2 70^o$

5. The equations of two lines are given, but the coefficient of y of one of the equations is illegible. The equations are $3x+4y=23$ and $2x+ay=0$. On a graph, it is shown that the two lines intersect at the point $(5,2)$. What is the value of *a?*

A. -5 B. 6 C. 8

D. -3 E. -4

6. Suppose $\log_5(\frac{x}{y})=2$ and $\log_5(xy)=6$. Find $x-y$.

A. 144 B. 225 C. 400

D. 500 E. 600

7. AB and CD are chords of a circle and meet at E. It is given that the lengths of AE, BE, and CE are 4, 10, and 2, respectively. What is the length of CD?

A. 14 B. 20 C. 22

D. 24 E. 28

8. Consider the four numbers $w=\ln(e)$, $x=\ln\ln(e\cdot e)$, $y=\ln\ln\ln(e\cdot e\cdot e)$, $z=\ln\ln(e+e+e)$. Which of the following lists these four numbers in increasing order?

A. w, x, y, z B. y, x, z, w C. x, y, z, w

D. y, x, w, z E. x, y, w, z

9. What is the largest prime factor of $5^{99}+5^{100}+5^{101}$?

A. 3 B. 5 C. 7

D. 31 E. 101

10. Suppose $\tan\theta = 1/3$ and $\tan\psi = 2$. Simplify $y = \tan(\theta+\psi)$.

A. 7 B. 8 C. -4

D. -6 E. Cannot be determined from the given information

11. Reduce the following fraction: $\dfrac{a^3+a^2+a+1}{a^2-1}$.

A. $\dfrac{a^2+1}{a+1}$ B. $\dfrac{a^2+1}{a-1}$ C. $\dfrac{a^2+a}{a+1}$

D. a E. $\dfrac{a^3+a^2}{a-1}$

12. Ella has some quarters and dimes that total $7.05. If all the quarters magically turned into dimes and vice versa, she would have $6.60. How many coins does she have?

A. 26 B. 34 C. 37

D. 39 E. 42

13. Simplify $\tan\left(\sin^{-1}(3/7)\right)$.

A. $4/7$ B. $\dfrac{3\sqrt{5}}{7}$ C. $\dfrac{3\sqrt{10}}{20}$

D. 0.8 E. $\dfrac{10\sqrt{7}}{3}$

14. Pocahontas shoots an arrow at you and then rides forward for 10 seconds at 9 feet per second and then shoots another arrow. If both arrows fly at 90 feet per second and both whiz by your head, how much time is there, in seconds, between the arrows that fly by your head?

A. 1 B. 6 C. 8

D. 9 E. 10

15. In how many orders can seven kids sit on three benches (each bench must have at least one kid)?

A. 5040 B. 50400 C. 64000

D. 68700 E. 75600

16. The sum of the solutions for the equation $\frac{7}{x-3}+\frac{6}{x+2}=2$ is

A. 6 B. 7/3 C. 15/2

D. 10 E. 23/5

17. What is the sum of the coordinates of the point of intersection of the system: $3x+4y=82,\quad 5x+8y=154$?

A. 20 B. 23 C. 27

D. 32 E. 40

18. Your ship sends messages by hoisting flags on a line. You have four distinct flags. Assuming a message must have at least one flag, how many different messages can you send?

A. 64 B. 360 C. 680

D. 780 E. 1728

19. In the x-y plane, the solution set to $|x+y|=2$ consists of two connected components. What is the shortest distance between these two components?

A. 1 B. 2 C. $\sqrt{2}$

D. $2\sqrt{2}$ E. $\sqrt{5}$

20. An equiangular hexagon has sides that alternate 2, 1, 2, 1, 2, 1. What is the area of the hexagon?

A. $4\sqrt{3}$ B. $\frac{5\sqrt{3}}{2}$ C. $\frac{13\sqrt{3}}{4}$

D. $\frac{55\sqrt{3}}{4}$ E. 6

21. Five coins are flipped onto the floor. Four of the them are randomly picked up and observed to be heads. What is the probability that all of the coins are heads?

A. 1/2 B. 1/3 C. 1/4

D. 1/5 E. 1/6

22. At one end of a see-saw, 10 feet from the fulcrum, sits a 200 pound man. Two 150 pound men want to sit on the other side, two feet apart, and balance the see-saw. How far, in feet, from the fulcrum should the closer man be?

A. 5 B. 6 C. 7

D. $5\frac{2}{3}$ E. $6\frac{1}{2}$

23. A rectangular room with North-South and East-West walls will have a stripe painted on the floor from a point on the East side, 1/3 of the way from the South to the North ends, to a point on the West side, 1/3 of the way from the North to the South ends. The length of this stripe and the side lengths of the room itself should be an integer number of units. From East to West the room is 10 units wide. What is the smallest possible North-South length of the room?

A. 10 B. 60 C. 72

D. 144 E. 360

24. A square and an equilateral triangle have the same area. The side length of the square is 5. What is the side length of the triangle?

A. $5\sqrt{3}$ B. $10\sqrt{3}$ C. $8\sqrt[3]{9}$

D. $\frac{10\sqrt[4]{27}}{3}$ E. $\frac{25\sqrt[4]{3}}{9}$

25. Suppose ABCD is a square, with side-length 1. The midpoints of BC and CD are Q and R. respectively. AQ and BR meet at P. What is the area of triangle BPQ?

A. 1/20 B. 1/10 C. 5/18

D. $\sqrt{5}/20$ E. $3\sqrt{3}/10$