

CALIFORNIA STATE UNIVERSITY, BAKERSFIELD

Lee Webb Math Field Day 2014

Team Medley, Junior-Senior Level

Each correct answer is worth ten points. Answers require justification. Partial credit may be given. Unanswered questions are given zero points.

You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the proctor. Multiple solutions to the same problem will invalidate each other.

Elegance of solutions may affect score and may be used to break ties.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

- (a) In Springfield, all streets run North-South or East-West. How many ways are there to get from Jimmy's house to Timmy's house? Both houses are at intersections and Timmy's house is 5 blocks to the north and 5 blocks to the east of Jimmy's.

(b) How many of these ways stay north of the direct, "as the crow flies," route?
- How many strings of 10 vowels can be made, if every vowel (a,e,i,o,u) has to be used at least once?
- Ella and Daniel have made up a game with small stones (they are so deprived – these are the only toys they have). The stones are placed in a pile and they take turns removing stones. At each turn, a player either removes one stone from the pile or removes half the stones (rounding up – if necessary) from the pile. The player who takes the last stone wins. Ella will go first (she always goes first since she's younger). Daniel says, "I get to decide how many stones go in the pile, since you can always win with some numbers of initial stones." Of which numbers is Daniel speaking?
- Prove the formula: $\pi = 16 \tan^{-1}\left(\frac{1}{5}\right) + 4 \tan^{-1}\left(\frac{1}{239}\right)$.
- Describe the set of all points that are equidistant from two circles. The circles have radii 1 and 3 and are centered at (0,0) and (6,0), respectively. Repeat in the case that the second circle is centered at (2,0).
- Parallel lines are spaced 2 units apart on a large board. A side-length 1 square is placed randomly on the board. What is the probability that a part of the square touches one of the parallel lines?