Chapter 1:Introduction to Design of Experiments

Introduction to DOE

- An **experiment** is a test or a series of tests.
- Experiments are used widely in the engineering and scientific world
  - Process characterization & optimization
  - Evaluation of material properties
  - Product design & development
  - Hardening process
  - Pharmaceuticals industries

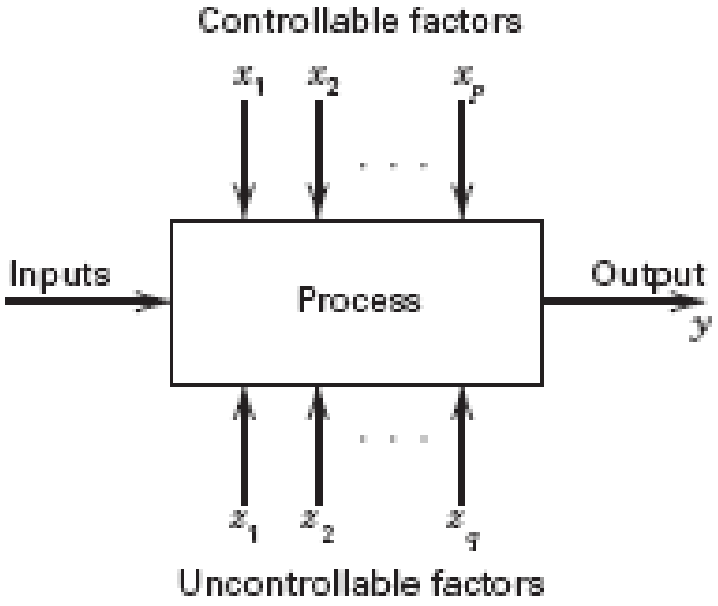
Theoretical (Mechanistic) and Empirical Models

- Some systems are understood well enough such that mathematical models exist that accurately characterize the system.
  - Equations from thermodynamics.
  - Electrical circuit equations.
  - Queuing equations.
  - …

Theoretical (Mechanistic) and Empirical Models

- Many (if not most) real systems are extremely complicated (from an analysis perspective) and operate in “noisy” unpredictable environments.
  - Simulation can sometimes be utilized.
- Empirical models are constructed from observation and collection of data from the system.
- Doing this correctly and efficiently is the objectives of DOE.

Engineering Experiments



■ FIGURE 1.1 General model of a process or system

Example

We have conducted a fixed effects model 2^3 factorial experiment to analyze the effects of various factors on popping microwaveable popcorn. Three different factors with two levels each were to be looked at.

- Response Variable: number of unpopped kernels
- Factors: A) Duration of cooking time: (1.75 minutes, 1.33 minutes);
  B) Powel level of the microwave: (full power, half power);
  C) Brand of popcorn: Top shelf (Pop Weaver), Bottom shelf (Trails End)

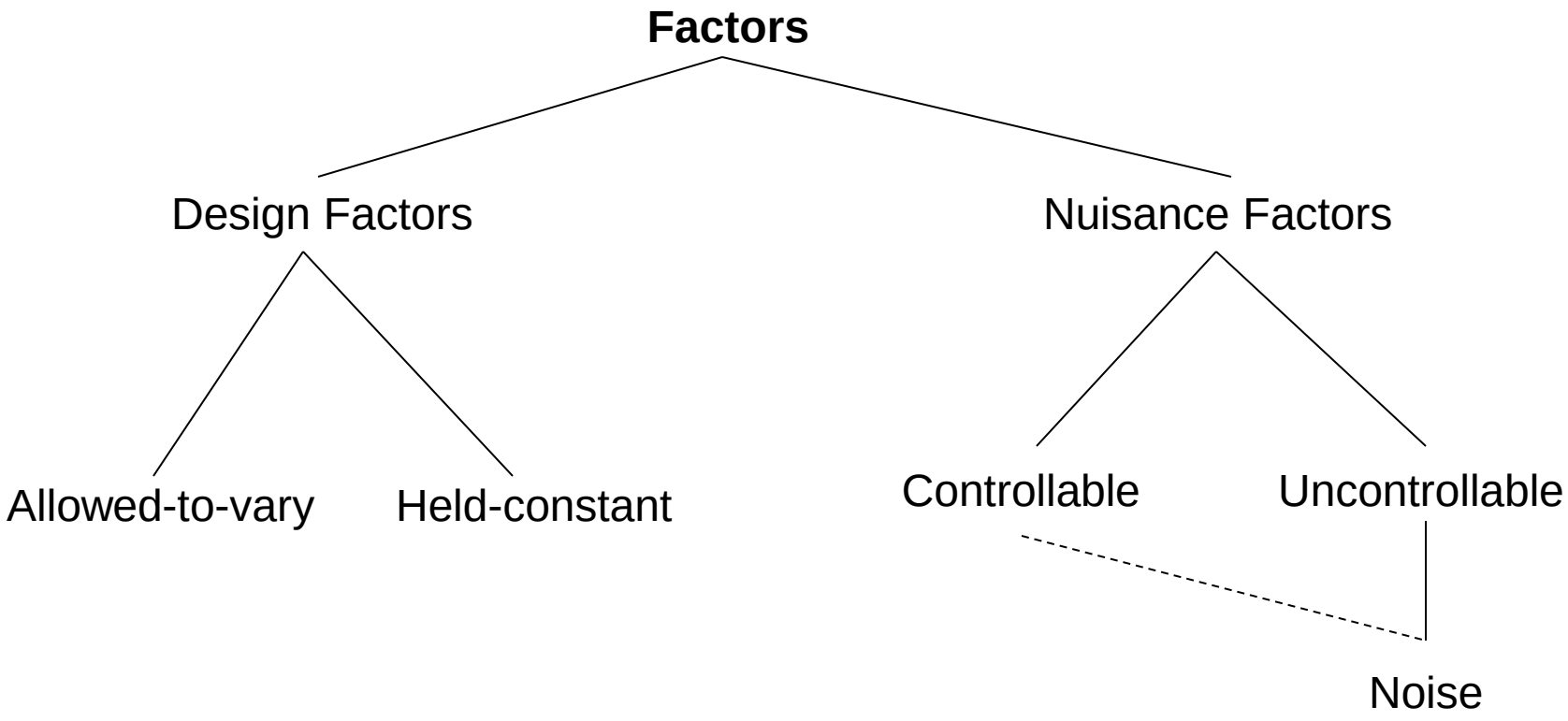
We conducted a total of 2 replicates for each treatment combination. This resulted in a total of 16 runs.

Nomenclature	Description
1	1.33 minutes, half power, bottom shelf (Run 1)
1	1.33 minutes, half power, bottom shelf (Run 2)
a	1.75 minutes, half power, bottom shelf (Run 1)
a	1.75 minutes, half power, bottom shelf (Run 2)
b	1.33 minutes, full power, bottom shelf (Run 1)
b	1.33 minutes, full power, bottom shelf (Run 2)
ab	1.75 minutes, full power, bottom shelf (Run 1)
ab	1.75 minutes, full power, bottom shelf (Run 2)
c	1.33 minutes, half power, top shelf (Run 1)
c	1.33 minutes, half power, top shelf (Run 2)
ac	1.75 minutes, half power, top shelf (Run 1)
ac	1.75 minutes, half power, top shelf (Run 2)
bc	1.33 minutes, full power, top shelf (Run 1)
bc	1.33 minutes, full power, top shelf (Run 2)
abc	1.75 minutes, full power, top shelf (Run 1)
abc	1.75 minutes, full power, top shelf (Run 2)

DOE Terminology

- Inputs/Experimental Setup
  - Factors
    - Treatments and levels
    - Fixed and random factors.
- Outputs/Results
  - Response variable
  - Interaction
  - Effect
- Fundamental DOE Principles
  - Replication
  - Randomization
  - Blocking

Inputs/Experimental Setup



Inputs/Experimental Setup

- Factors
  - Treatments and levels
    - Terminology for specific values of a factor
  - Fixed and random factors.
    - Fixed – specific treatment values are selected and of interest.
    - Random – individual levels are randomly selected from a population.

Outputs/Results

- Response variable
- Effect
  - Interaction

Fundamental DOE Principles

- Replication
  - Sample size (improving precision of estimation, estimation of error or background noise).
  - More precision requires more work.
- Randomization
  - Running the trials in an experiment in random order.
    - Both the allocation of the experimental material and the order in which the individual runs of the experiment are to be performed are randomly determined. (Use random number generators)
    - Sometimes, it is difficult to have complete randomization (e.g. Temperature setting), see Chapter 14
  - Balance effects of uncontrollable variables.
- Blocking (A restriction on randomization)
  - A method to isolate variability contributed by level changes of specific factors.
  - Randomized block design

Different Types of Experimentation Strategy

- "Best-guess" experiments
  - Quite common.
  - Utilizes detailed knowledge of the situation.
  - Disadvantages: might not produce the desired outcome; time consuming.
- One-factor-at-a-time experiments
  - Holding all but one factor constant. Repeat systematically.
  - It fails to consider any possible interaction between the factors.
- Statistically designed experiments
  - Vary factors together in a specific manner – factorial experiments.

Factorial Experiment

- In a factorial experiment, all possible combinations of factor levels are tested.
- Golf example
  - Type of driver (oversized/regular sized)
  - Type of ball (balata/three piece)
  - Walking vs. riding
  - Type of beverage
  - Weather (cool/hot)
  - Type of golf spike (metal/soft)
- Goal: lower the score

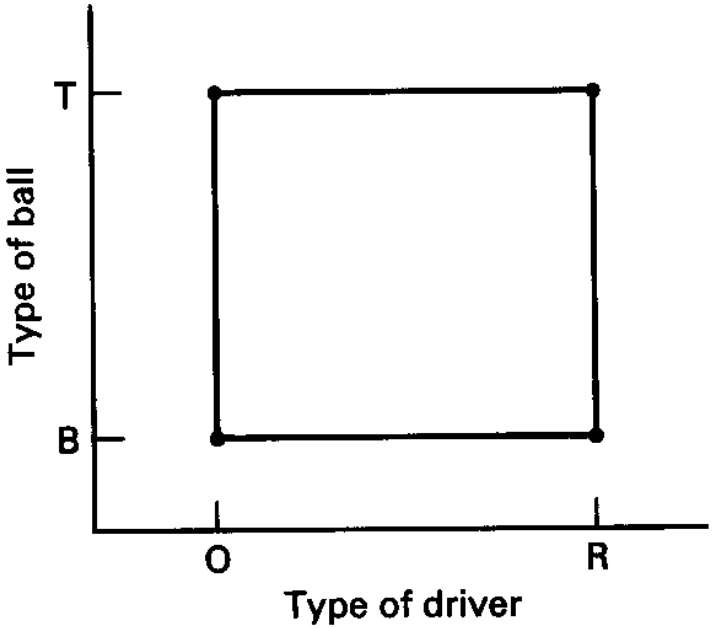
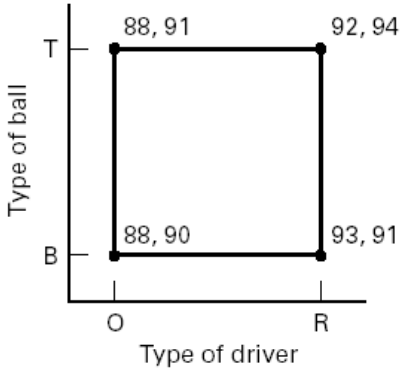
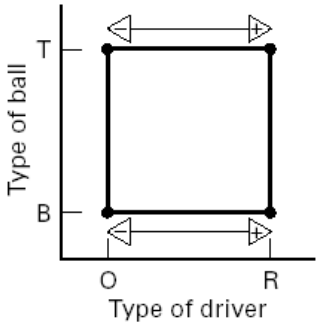


Figure 1-4 A two-factor factorial experiment involving type of driver and type of ball.

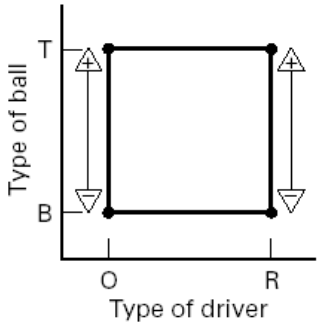
2^2 Factorial Design (with 2 replicates)



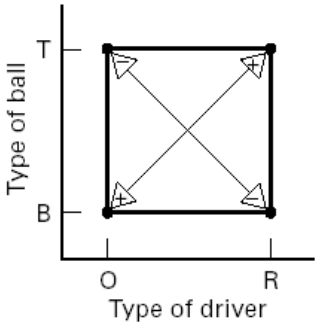
(*a*) Scores from the golf experiment



(*b*) Comparison of scores leading to the driver effect



(*c*) Comparison of scores leading to the ball effect



(*d*) Comparison of scores leading to the ball-driver interaction effect

Figure 1-5 Scores from the golf experiment in Figure 1-4 and calculation of the factor effects.

$$\text{Driver effect} = \frac{92 + 94 + 93 + 91}{4} - \frac{88 + 91 + 88 + 90}{4}$$
$$= 3.25$$

$$\text{Ball effect} = \frac{88 + 91 + 92 + 94}{4} - \frac{88 + 90 + 93 + 91}{4}$$
$$= 0.75$$

$$\text{Ball–driver interaction effect} = \frac{92 + 94 + 88 + 90}{4} - \frac{88 + 91 + 93 + 91}{4}$$
$$= 0.25$$

2^3 Factorial Designs

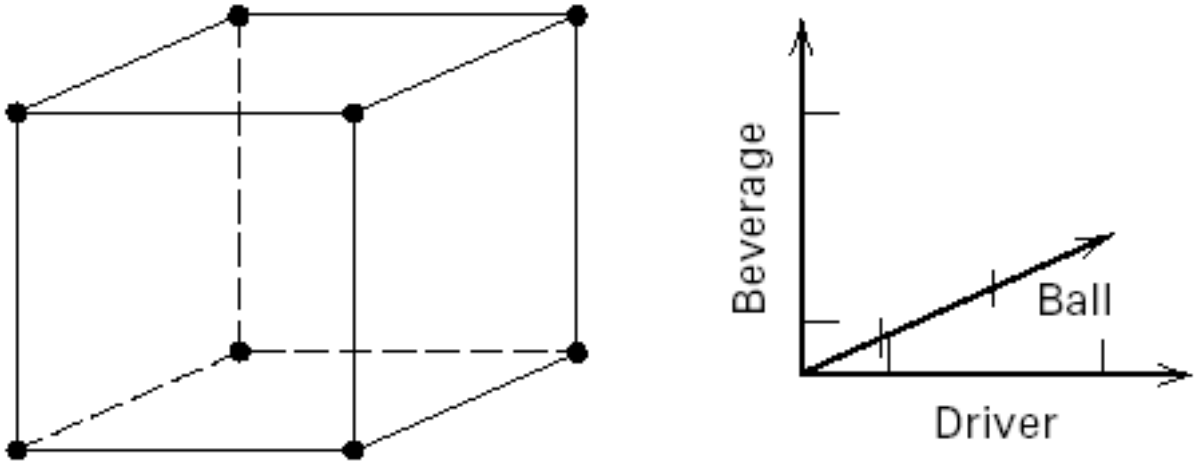


Figure 1-6 A three-factor factorial experiment involving type of driver, type of ball, and type of beverage.

2^4 Factorial Designs

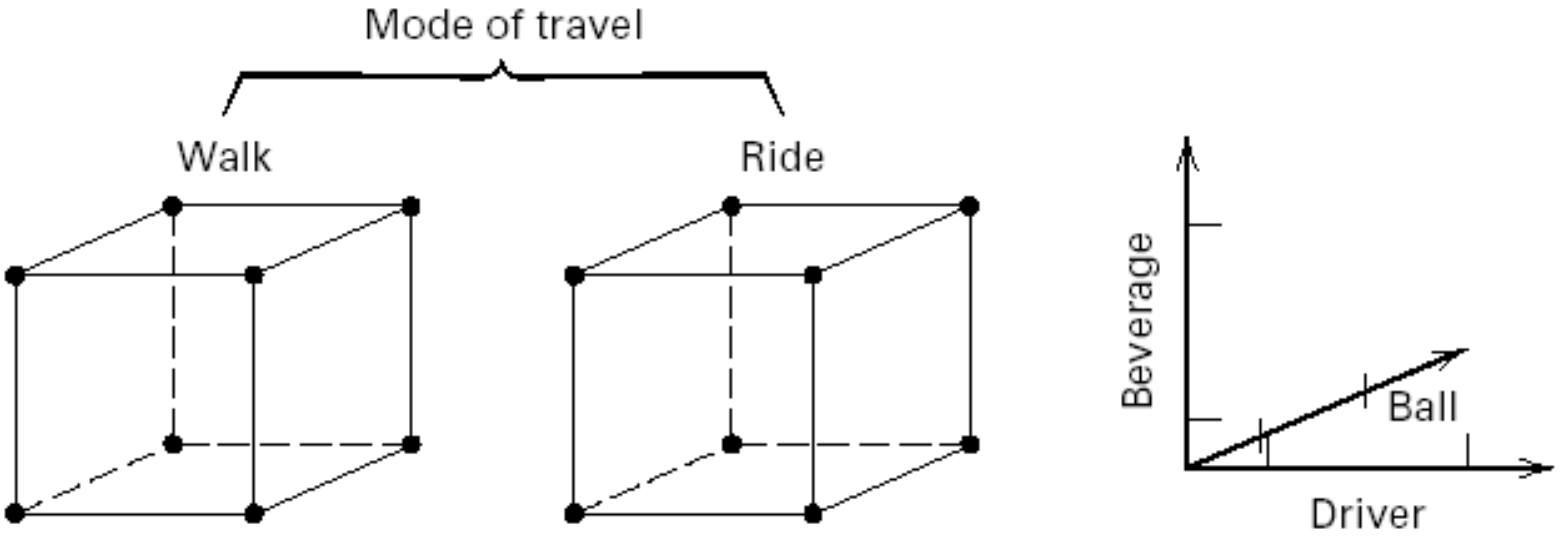
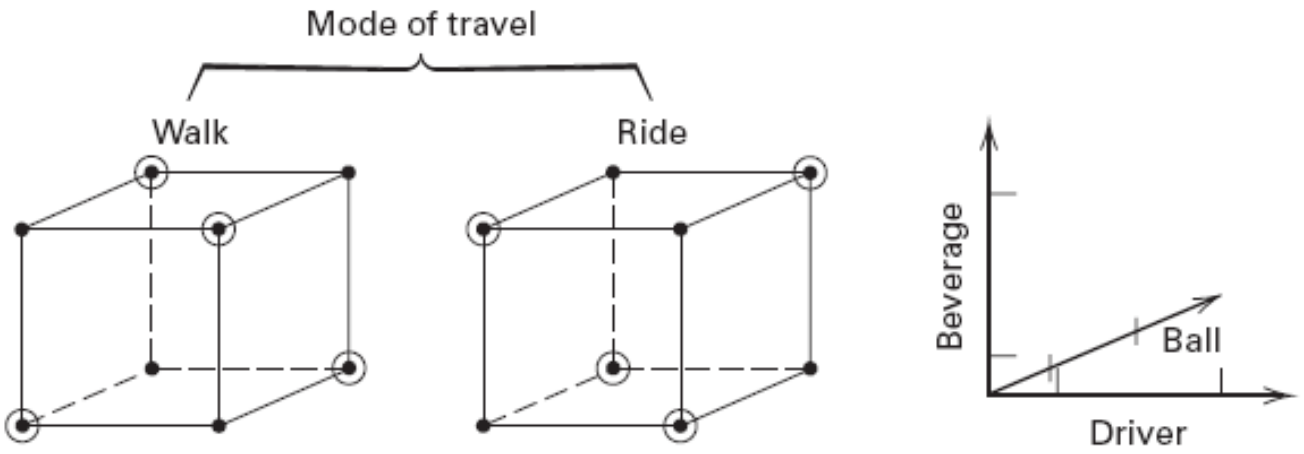


Figure 1-7 A four-factor factorial experiment involving type of driver, type of ball, type of beverage, and mode of travel.

2^k Factorial Designs (k>4):

Fractional Factorial Experiment:

when there are 4 or more factors. It is usually unnecessary to run all possible combinations of factor levels. (Chapter 8 and 9)



■ FIGURE 1.8 A four-factor fractional factorial experiment involving type of driver, type of ball, type of beverage, and mode of travel

Experimental Design - Outline

- Probability & statistics (Already discussed)
  - Confidence intervals and hypothesis tests.
  - Comparisons - Single factor, two level experiments.
- Intro to ANOVA – Single factor fixed effects model.
- Multifactor experiments – fixed factors
- Blocking
- 2^k experiments
- Experiments with random factors
- Nested factors